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Topic: Interacting and Non-Interacting System, Closed loop system, Block Diagram, Controllers, Transient Response of closed loop system.

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Example → Non-interacting System.

Modelling Eq.

Tank 1

$$A_1 \frac{dh_1}{dt} = f_i - f_1$$

$$f_1 = \frac{h_1}{R_1}$$

$$h_1 = f_1 R_1$$

$$A_1 R_1 \frac{df_1}{dt} = f_i - f_1$$

$$\tau_{p_1} \frac{df_1}{dt} = f_i - f_1 \quad \text{--- (1)}$$

At S.S.

$$\tau_{p_1} \frac{df_{1ss}}{dt} = 0 = f_{iss} - f_{1ss} \quad \text{--- (2)}$$

(1) - (2)

$$\tau_{p_1} \frac{d(f_1 - f_{1ss})}{dt} = (f_i - f_{iss}) - (f_1 - f_{1ss})$$

$$\tau_{p_1} \frac{dF_1}{dt} = F_i - F_1$$

$$\tau_{p_1} [sF_1(s) - \cancel{F_1(0)}] = F_i(s) - F_1(s)$$

$$\frac{F_1(s)}{F_i(s)} = \frac{1}{(\tau_{p_1} s + 1)} = G_1(s) \quad \text{--- (I)}$$

$$\tau_{p_1} = A_1 R_1 \quad \& \quad K_{p_1} = 1$$

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Tank 2

$$A_2 \frac{dh_2}{dt} = f_1 - f_2$$

$$f_2 = h_2/R_2$$

$$A_2 \frac{dh_2}{dt} = f_1 - \frac{h_2}{R_2} \quad \text{--- (i)}$$

At S.S.

$$A_2 \frac{dh_{2ss}}{dt} = 0 = f_{1ss} - \frac{h_{2ss}}{R_2} \quad \text{--- (ii)}$$

(i) - (ii)

$$A_2 \frac{dH_2}{dt} = F_1 - \frac{H_2}{R_2}$$

$$A_2 [sH_2(s) - \cancel{H_2(0)}] = F_1(s) - \frac{1}{R_2} H_2(s)$$

$$\left(A_2 s + \frac{1}{R_2}\right) H_2(s) = F_1(s)$$

$$\frac{H_2(s)}{F_1(s)} = \frac{R_2}{(A_2 R_2 s + 1)}$$

$$\frac{H_2(s)}{F_1(s)} = \frac{k_{p_2}}{(\tau_{p_2} s + 1)} = G_2(s) \quad \text{--- (II)}$$

$$k_{p_2} = R_2, \quad \tau_{p_2} = A_2 R_2$$

Now,

$$G_0(s) = G_1(s) \times G_2(s)$$

$$G_0(s) = \frac{F_1(s)}{F_i(s)} \times \frac{H_2(s)}{F_1(s)}$$

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$$\boxed{G_0(s) = \frac{k_{p_1}}{(\tau_{p_1}s+1)} \times \frac{k_{p_2}}{(\tau_{p_2}s+1)} = \frac{H_2(s)}{F_i(s)}}$$

$$\frac{H_2(s)}{F_i(s)} = \frac{k_{p_1}k_{p_2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

For a step change of unit mag. in f_i

i.e. $f_i(t) = 1 \longrightarrow F_i(s) = 1/s$

$$H_2(s) = \frac{k_{p_1}k_{p_2}}{s(\tau_{p_1}s+1)(\tau_{p_2}s+1)}$$

$$H_2(s) = k_{p_1}k_{p_2}\left[\frac{1}{s(\tau_{p_1}s+1)(\tau_{p_2}s+1)}\right]$$

Taking Inverse Laplace

$$L^{-1}\{H_2(s)\} = k_{p_1}k_{p_2}L^{-1}\left[\frac{A}{s} + \frac{B}{(\tau_{p_1}s+1)} + \frac{C}{\tau_{p_2}s+1}\right]$$

On Solving

$$\boxed{H_2(t) = k_{p_2}\left[1 + \frac{1}{(\tau_{p_1}\tau_{p_2})}\left\{\tau_{p_1}e^{-t/\tau_{p_1}} - \tau_{p_2}e^{-t/\tau_{p_2}}\right\}\right]}$$

as $k_{p_1} = 1$

Conclusion:

① Resp. of Non-infer. Sys. is initially slow & then it picks up speed. (S-shaped Curve).

② On comparing curve 2 & 3, we observe that change in Resp. of 4 non-int. sys. is more slower than that of 2-Non-int. sys.

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(30) Consider two non-interacting tanks in series as shown in fig. Water enters tank 1 at 'q' cm^3/s and drains down to tank 2 by gravity at a rate of $K\sqrt{h_1}$ (cm^3/s). Similarly water drains from tank 2 by gravity at a rate of $K\sqrt{h_2}$ (cm^3/s) where h_1 and h_2 represent level of tank 1 and tank 2 respectively (see fig). Drain value constant $K = 4\ cm^{2.5}/s$ and cross-sectional Areas of two tanks are $A_1 = A_2 = 28\ cm^2$.

At steady state operation, the water inlet flow rate is $q_{ss} = 16\ cm^3/s$. The transfer function relating the deviation variables (cm) to flow rate q (cm^3/s) is

Sol'n, $G(s) = \dfrac{H_2(s)}{q(s)} = \dfrac{R_2}{(\tau_1 s+1)(\tau_2 s+1)}$

$R_2 = \dfrac{1}{q_2'(h_{ss})}$

$q_2 = K\sqrt{h_2}$

$q_2'(h_{ss}) = \dfrac{K}{2\sqrt{h_{ss}}}$

at S.S.

$q = q_1 = 16\ cm^3/s$

~~$q_2 = q_1 = 16\ cm^3/s$~~

$q_1(ss) = K\sqrt{h_{ss}}$

$4\ cm^{2.5}/s = 4\ cm^{2.5}\sqrt{h_{ss}}$

$h_{ss} = 16\ cm$

$\rightarrow q_2'(h_{ss}) = \dfrac{4\ cm^{2.5}/s}{2\sqrt{16\ cm}}$

$q_2'(h_{ss}) = \dfrac{4}{2\times 4}$

$R_2' = 2$

Now, $R_2 = \dfrac{1}{q_2(h_{ss})}$

$R_2 = 2$

$\tau_2 = A_2 \times R_2 = 28 \times 2 = 56$

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$$R_1 = \frac{1}{q_1'(h_{ss})} \qquad q_1 = k\sqrt{h_1}$$

$$q_1'(h_1) = \frac{k}{2\sqrt{h_{ss}}} = \frac{\cancel{k}}{2 \times \cancel{k}}$$

$$R_1 = \frac{1}{\frac{1}{2}} = 2$$

$$\tau_1 = A_1 \times R_1 = 28 \times 2 = 56$$

$$G(s) = \frac{H_2(s)}{Q(s)} = \frac{2}{(56s+1)^2}$$

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Example: INTERACTING SYSTEM

Tank 1

$$A_1 \frac{dh_1}{dt} = f_i - f_1$$

$$f_1 = \frac{(h_1 - h_2)}{R_1}$$

$\because$ Liq. level $h_1 > h_2$.

$$A_1 R_1 \frac{dh_1}{dt} + h_1 - h_2 = R_1 f_i \quad \text{------} \quad (1)$$

Tank 2

$$A_2 \frac{dh_2}{dt} + f_2 = f_1$$

$$A_2 \frac{dh_2}{dt} + \frac{h_2}{R_2} = \frac{(h_1 - h_2)}{R_1} \qquad \left\{ \because f_2 = \frac{h_2}{R_2} \right\}$$

$$A_2 R_2 \frac{dh_2}{dt} + \left(1 + \frac{R_2}{R_1}\right) h_2 - \frac{R_2}{R_1} h_1 = 0 \quad \text{---} \quad (2)$$

In terms of Dev. Var.

Tank 1

$$A_1 R_1 \frac{dH_1}{dt} + H_1 - H_2 = R_1 F_i(t)$$

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Taking the Laplace

$$A_1 R_1 \{ s H_1(s) - \cancel{H_1(0)}^{0} \} + H_1(s) - H_2(s) = R_1 F_i(s) \quad \text{- - - - - - - (3)}$$

Take 2

$$A_2 R_2 \frac{dH_2}{dt} + \left(1 + \frac{R_2}{R_1}\right) H_2 - \frac{R_2}{R_1} H_1 = 0$$

$$A_2 R_2 \{ s H_2(s) - \cancel{H_2(0)}^{0} \} + \left(1 + \frac{R_2}{R_1}\right) H_2(s) = \frac{R_2}{R_1} H_1(s)$$

$$\left\{ A_2 R_2 s + \left(1 + \frac{R_2}{R_1}\right) \right\} H_2(s) - \frac{R_2}{R_1} H_1(s) = 0 \quad \text{- - - - - (4)}$$

From eq. (3)

$$\cancel{H_1(s) = R_1}$$

$$\{ A_1 R_1 s + 1 \} H_1(s) = H_2(s) + R_1 F_i(s)$$

$$H_1(s) = \frac{1}{(A_1 R_1 s + 1)} H_2(s) + \frac{R_1}{(A_1 R_1 s + 1)} F_i(s)$$

put $H_1(s)$ in eq. (4)

$$\cancel{A_2 R_2 s H_2(s) +}$$

$$\left\{ A_2 R_2 s + \left(1 + \frac{R_2}{R_1}\right) \right\} H_2(s) = \frac{R_2}{R_1} \left\{ \frac{H_2(s)}{(A_1 R_1 s + 1)} + \frac{R_1}{(A_1 R_1 s + 1)} F_i(s) \right\}$$

$$\left\{ \tau_{p_2} s + 1 + \frac{R_2}{R_1} - \frac{R_2}{R_1 (\tau_{p_1} s + 1)} \right\} H_2(s) = \frac{R_2}{(\tau_{p_1} s + 1)} F_i(s)$$

$$\frac{\{ (\tau_{p_1} s + 1) \tau_{p_2} s R_1 + R_1 (\tau_{p_1} s + 1) + R_2 (\tau_{p_1} s + 1) - R_2 \}}{R_1 \cancel{(\tau_{p_1} s + 1)}} H_2(s) = \frac{R_2}{\cancel{(\tau_{p_1} s + 1)}} F_i(s)$$

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$$\boxed{G(s) = \frac{H_2(s)}{F_1(s)} = \frac{R_2}{\tau_{p_1}\tau_{p_2}s^2 + (\tau_{p_1}+\tau_{p_2}+A_1R_2)s + 1}}$$

Overall T.F. of Interacting Sys.

Non-Interacting Sys.

$$\frac{H_2(s)}{F_1(s)} = \frac{k_{p_2}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1)} = \frac{R_2}{\tau_{p_1}\tau_{p_2}s^2 + (\tau_{p_1}+\tau_{p_2})s + 1}$$

On Comparing

1 extra term A_1R_2 is present in overall $G(s)$ of Interacting System.

$A_1R_2 \rightarrow$ Interaction Factor

Conclusion →

(1) IInd Order Response by connecting 2 Cap. in Series.

(2) $$P = \frac{-(\tau_{p_1}+\tau_{p_2}+A_1R_2) \pm \sqrt{(\tau_{p_1}+\tau_{p_2}+A_1R_2)^2 - 4\tau_{p_1}\tau_{p_2}}}{2\tau_{p_1}\tau_{p_2}}$$

Roots

$$(\tau_{p_1}+\tau_{p_2}+A_1R_2)^2 - 4\tau_{p_1}\tau_{p_2} > 0$$

i.e. 2 Distinct real roots.

$\therefore \zeta > 1 \longrightarrow$ Overdamped Response for Interacting Sys. always.

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Hardware Elem. of Control Syst.

Example → Heating Tank System.

Obj. → Control temp. within Reactor.

Control Pair

CV	MV
T	F_{st} (we choose)

∴ Closed loop sys. is obt. from open loop sys. by applying a control scheme

INTRODUCTION TO FEEDBACK CONTROL :

Development of Block diag. of closed loop process →

Example: Heating Tank System.

Control obj :- $T = T_{ss}$

Assumption : $f_i = f$ {same & Constant}

Control Pair :

CV	MV	LV
T	f_{st}	T_i

open loop Heating tank Sys.

→ We have assume that $f_i = f$, & these indicate there is no variation in h i.e. $(h = h_{ss})$ → Constant

$\varepsilon = T_{sp} - T_m$

By the introduction of Controller, loop is get closed.

closed loop Heating Tank Sys.

Block diag. of open loop process →

M(s) → output from final control element.

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Block Diag. of closed loop sys. ⇒

$Y_m = T_m$ (measured temp.)

W.C.R.

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BLOCK DIAG. OF SIMPLE CONTROLL SYSTEM

Set Point Variable → It is the desired value of a Controlled variable.

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Load Variable →

It refer to a change in any variable that may cause the controlled variable of the process to change.

Feedback →

As the meas. value of the controlled variable is returned or feedback to the system.

Negative feedback v/s Positive Feedback →

We are using the diff. of set value of measured value, these is known as negative feedback. If we measure the addition of set point & the measured value then it is known as positive feedback. Positive feedback makes the system unstable and ∴ not used in chemical process industry.

SERVO V/S REGULATORY

SERVO →

No change in the load but set point changes.

REGULATORY →

No change in the set point but load variable change.

In the chemical process industry, our aim

is to maintain steady state and to controll the value of controlled variable at some constant desired value. Therefor we keep the set point ~~change~~ same But disturbance may change, that's why Regulatory mode follows in chemical process industry.

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BLOCK DIAG. FOR a Generalised Closed-Loop System:

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For Regulator Mode

$Y_{sp}(s) = 0$ if change in disturbance only

$$\frac{Y(s)}{d(s)} = \frac{G_d}{1 + G_p G_c G_F G_m}$$

For Servo Mode

$d(s) = 0$ if change in set point value only.

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{G_c G_F G_p}{1 + G_p G_c G_F G_m}$$

$$\boxed{Y = \frac{G_d}{1 + G_p G_c G_F G_m} d + \frac{G_p G_c G_F}{1 + G_c G_p G_F G_m} Y_{sp}}$$

Principle of Superposition

$$\boxed{Y = G_{sp}(s) Y_{sp} + G_{dc}(s) d}$$

$$G_{sp}(s) = \frac{G_p G_c G_F}{1 + G_p G_c G_F G_m}$$

↓ closed loop set point transfer funct

$$G_{dc}(s) = \frac{G_d}{1 + G_p G_c G_F G_m}$$

↓ closed loop disturbance transfer funct

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* Kinetic of closed loop is greater than that of open loop.

Open-Loop Transfer function →

$$\Rightarrow G_{OL}(s) = G_p G_c G_F G_m$$

$$\boxed{Y = \frac{G_p G_c G_F}{1 + G_{OL}} Y_{sp} + \frac{G_d}{1 + G_{OL}} d}$$

Closed-Loop Transfer function →

$$\Rightarrow G_{CL}(s) = \frac{\text{T.F. in forward Path}}{1 + G_{OL}(s)}$$

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#. Dynamic Behaviour of Closed Loop Process:

Block Diag. of Closed-loop System

Process: $y(s) = G_p(s)\, m(s) + G_d(s)\, d(s)$.

Meas. Device: $y_m(s) = G_m(s)\, y(s)$.

Controller: $\epsilon(s) = y_{sp}(s) - y_m(s)$ ---- Comparator

$C(s) = G_c(s)\, \epsilon(s)$ ---- Controller.

F.C.E: $m(s) = G_f(s)\, C(s)$.

$$m(s) = G_f(s)\, G_c(s)\, \epsilon(s)$$

$$= G_f(s)\, G_c(s) \left[y_{sp}(s) - y_m(s) \right]$$

$$m(s) = G_f(s)\, G_c(s) \left[y_{sp}(s) - G_m(s)\, y(s) \right]$$

Now.

$$y(s) = G_p(s)\, m(s) + G_d(s)\, d(s)$$

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$$y = G_p G_f G_c [y_{sp} - G_m y] + G_d d$$

$$y = G_p G_c G_f y_{sp} - G_p G_c G_f G_m y + G_d d$$

$$\boxed{y = \frac{G_p G_c G_f}{(1 + G_p G_c G_f G_m)} y_{sp} + \frac{G_d}{(1 + G_p G_c G_f G_m)} d}$$

It ~~represent~~ provide effect of y_{sp} on y.

It provide effect of d on y.

..... closed loop T.F. (CLTF).

If $G_p G_c G_f = G$

then

$$y = \frac{G}{1 + G G_m} y_{sp} + \frac{G_d}{1 + G G_m} d \quad \checkmark$$

$$y = G_{sp} y_{sp} + G_{load} d$$

where

$$G_{sp} = \frac{G}{1 + G G_m}$$

$$G_{load} = \frac{G_d}{1 + G G_m}$$

$$y = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} y_{sp} + \frac{G_d}{1 + G_p G_c G_f G_m} d$$

- Use this T.F. usually in the analysis of diff. features.

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- Control performance is usually investigated by performing 2 test, one is servo test & Regulatory test.

<u>Servo Test:</u> $d = 0$

$$y = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} \cdot y_{sp}$$

servo problem / set point tracking.

<u>Regulatory Test:</u> $y_{sp} = 0$

$$y = \frac{G_d}{1 + G_p G_c G_f G_m} \cdot d$$

Disturbance Rejection performance.

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Example → Formation of Closed Loop Diag. for a liquid level System:

Obj: $h = h_{ss}$

Control Pair:

CV	MV	LV
h	f_o	f_i

DPC → Differential Press. Cell

Process:

$$A\frac{dh}{dt} = f_i - f_o$$

$$A s\, h(s) = f_i(s) - f_o(s) \quad \text{------ Laplace Transfor}$$

$$h(s) = \frac{1}{As} f_i(s) - \frac{1}{As} f_o(s) \quad \text{.... Liquid tank}$$

~~General form~~

$$y(s) = G_p(s)\, m(s) + G_d(s)\, d(s) \quad \text{.... General form}$$

$m \to f_o$ & $d \to f_i$

So, $G_p(s) = -\frac{1}{As}$, $G_d(s) = \frac{1}{As}$

Measuring Device:

$$\Delta P \rightarrow \boxed{DPC} \rightarrow h_m$$

$\Delta P \propto h$ $\qquad$ $\Delta P = \beta h$

Considering 2nd ord. dynamic of DPC.

$$\tau^2 \frac{d^2 h_m}{dt^2} + 2\tau\zeta \frac{dh_m}{dt} + h_m = k_p \Delta P = k_p \beta h$$

$$G_m(s) = \frac{h_m(s)}{h(s)} = \frac{k_p \beta}{(\tau^2 s^2 + 2\tau\zeta s + 1)} \quad \text{.... Laplace Transfor}$$

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Controller :

PI Controller . . $G_c(s) = k_c\left(1 + \frac{1}{\tau_I s}\right)$

F.C.E :

Assuming 1st ord. dynamics of control valve.

$$G_f = \frac{k_v}{(\tau_v s + 1)}$$

$k_v \rightarrow$ Gain of control valve

$\tau_v \rightarrow$ Time constant of control valve.

Fig : Closed loop block diag. of liq. level system

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Hardware involve in M.T.S.

① **Process**: Phys. & chemical oper.

② **Measuring Instr. or Sensors**: Use to meas. LV, CV, Secondary output.

Variable	Sensor (from notes of PDC.)

• Hg thermometer is not good b'coz measurement signal cannot be readily transmitted.

Liq. level → DP Cell (Diff. Press.)

③ **Transducer**: Measurements cannot be use until they converted to physical quant. (voltage, current) which can readily transmitted.

Measurement → Physical quant.

④ **Transmission line**: Carrying signals.

Sensor → Controller → Final Control element.

• Meas. signal sometimes very weak, in that case Transmission line equipped with amplifier.

⑤ **Controller**: • Receive meas. signal from sensor
• And then decide meas. action should be taken.

⑥ **Final Control Element**: Control action taken by controller is phy. implemented by final control element.

Eg:- Flow Control valve, Variable speed pump & Compressor etc.

⑦ **Recording Device**: Visualistic behaviour. (Video display unit).

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Types of Controllers :

$$\xrightarrow{E(s)} \boxed{\text{Controller}} \xrightarrow{C(s)}$$

1. Proportional Controller (P-only Controller).
2. Proportional-Integral Controller (PI-Controller).
3. Proportional-~~Der~~ Integral Derivative Controller (PID).

These are basic feedback controller.

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① P-Controller: $C(t) \propto \varepsilon(t)$

$C(t) = k_c \varepsilon(t)$

$C(t) = C_s + k_c \varepsilon(t)$

$k_c \rightarrow$ Proportional Gain of Controller.

$C_s \rightarrow$ Bias Signal of Controller

Bias signal is controller output, when there is no Error.

$C(t) = C_s$ {or $\varepsilon = 0$}.

$k_c \rightarrow$ Tuning parameter of controller, value of which we need to determine.

$C_s \rightarrow$ steady state controller output.

$\varepsilon \rightarrow$ Error

$C(t) = k_c \varepsilon(t)$

$C(s) = k_c \varepsilon(s)$

$$\boxed{G_c(s) = \frac{C(s)}{\varepsilon(s)} = k_c}$$

P-Controller exhibit no additional dynamic system, if the system is of 1st order. It remains 1st order. That is the reason why P-Controller is the fastest Controller.

P-Controller are sensitive towards error signal. For medium of large change in the signal Controller will produce desired output as the output is proportion to the error. But for a small change in error P controller is not useful as output produced is not desired.

To rectify these problem the gain of Controller can be increased such that the multi-plication of gain & error will produce desired output but it also make the Controller sensitive if the large change in Error comes then it will produce output which is undesired and makes the system unstable.

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PI Controller

$$\boxed{P = \underbrace{K_c \epsilon}_{P} + \underbrace{\frac{K_c}{\tau_I}\int_0^t \epsilon \, dt}_{I} + C_s}$$

P controller alone cannot be used for the small change in error signal. To rectify the problem integral action is added along with the proportional action.

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The integral action increase magnitude of error & combined effect of both to get desired ~~output~~ controller output.

$$\bar{C}(s) = K_c E(s) + \frac{K_c}{\tau_I} \frac{E(s)}{s}$$

$$\boxed{\frac{\bar{C}(s)}{\bar{E}(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right)}$$

By adding the I action, the dynamics of system increases by 1 as can be seen from G(s) of I controller, ∴ I action makes the system sluggish.

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τ_I → Integral time / Reset time. τ_I

τ_I is defined as the time taken by "I" Action to repeat the 'P' action.

Reciprocal of τ_I is known as reset rate where reset rate means how rapidly frequently P action repeated per unit time.

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(2.) PI - Controller:

$$C(t) = \underbrace{k_c\, \epsilon(t)}_{\text{P-Action}} + \underbrace{\frac{k_c}{\tau_I}\int_0^t \epsilon(t)\,dt}_{\text{I-Action}}$$

Taking Laplace

$$C(s) = k_c\, \epsilon(s) + \frac{k_c}{\tau_I}\,\frac{\epsilon(s)}{s}$$

$$\boxed{G(s) = \frac{C(s)}{\epsilon(s)} = k_c\left(1 + \frac{1}{\tau_I s}\right)}$$

(i) I-action eliminate the offset. (steady state error).

(ii) PI-Controller can eliminate the offset due to addition of I-action that means only P-Action cannot eliminate the offset.

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(ii) Due to addition of I-action, the overall response becomes more sluggish due to increase of order by 1.

Eg → I^{st} Order Sys. + PI Controller → IInd order System.

As increase in no. of capacities in series increases the sluggishness of response.

Hence addition of PI-controller to first order sys. will result in the more sluggishness in overall response.

(iii) Unit step change in $e(t)$.

$$e(t) = 1$$

$$E(s) = 1/s$$

$$C(s) = \frac{1}{s}\left(k_c + \frac{k_c}{\tau_I s}\right)$$

Inverse Laplace

$$\mathcal{L}\, c(t) = k_c + \frac{k_c}{\tau_I} t$$

$$\boxed{c(t) = (c_s + k_c) + \frac{k_c}{\tau_I} t}$$

This is behaviour of PI Controller → against a step change in error signal or the change in error signal introduced at $t=0$.

At $t=0$, we get a step change & at that $t=0$, controller output is (c_s+k_c).

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I-action →

$$I = \frac{k_c}{\tau_I}\int_0^t e(t)\,dt$$

τ_I → Integral time constant / Reset time.

$$C(t) = k_c + \frac{k_c t}{\tau_I}$$

if $t = \tau_I$ → $C(t) = k_c + k_c$

$C(t) = 2k_c$

Reset time is the time taken by I-action to Repeat initial proportional action change in its output.

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PI Controller

As long as the error is present, PI Controller keeps changing its output by integrating the error.

$$f_{st}(t) = f_{st_s} + k_c \epsilon(t) + \underbrace{\frac{k_c}{\tau_I}\int_0^t \epsilon(t)\,dt}_{?}$$

$$60\% = 50\% + 0 + 10\%$$

(as I-action eliminates the offset).

If we give step change in temp.
$25^\circ C \longrightarrow 20^\circ C$

then, there is a need to keep the steam valve 60% open.

$$\underbrace{\frac{k_c}{\tau_I}\int_0^t \epsilon(t)\,dt}_{} = 10\%$$

It is not zero, it remains constant at the last value (that means ϵ at last step is zero but the sum of previous errors ϵ provide 10%).

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PD CONTROLLER

$$\boxed{c(t) = \underbrace{k_c \epsilon}_{P} + \underbrace{k_c \tau_D \frac{d\epsilon}{dt}}_{D}}$$

$$\boxed{\frac{\bar{C}(s)}{\bar{\epsilon}(s)} = k_c(1 + \tau_D s)}$$

change for unit Ramp change Cof. response

$$\epsilon(s) = \frac{1}{s^2}$$

$$\bar{C}(s) = k_c\left(\frac{1}{s^2} + \frac{\tau_D}{s}\right)$$

$$c(t) = k_c(t + \tau_D)$$

$$\bar{C}(t) = k_c t + k_c \tau_D$$

$$\boxed{C(t) = k_c t + k_c \tau_D + C_s}$$

These eq. shows that when the ramp input is given to the system, the output will be ramp in nature that means kinetic of overall system remains same.

D action does not solve the problem of P action as it decreases the magnitude of error of makes it even more slow.

For the ramp input D action give step output

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For step input, output of D action is zero there means whether we are controlling by P or D has same means as D action has no use. Therefore PD controller is not used chemical process industries.

T_D is known as the Derivative time which is defined as by which D-action advances the P-action.

PD controller is faster in comparision to P-controller.

D-action is the derivative action & it defines the rate of change of the error signal. These rate of change will help the controller to achieve a head start in the output before the error enters the controller, which means D-action works in anticipatory manner. It will predict the error that may come in the future and applies the controll action necessary. This is the reason why D-action is fastest in compare to P-action and I-action.

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(3) PID Controller:

$$c(t) = \underbrace{k_c \epsilon(t)}_{P} + \underbrace{\frac{k_c}{\tau_I}\int_0^t \epsilon(t)\,dt}_{I} + \underbrace{k_c \tau_D \frac{d\epsilon(t)}{dt}}_{D} + C_s$$

$$C(t) = k_c\left\{\epsilon(t) + \frac{1}{\tau_I}\int_0^t \epsilon(t)\,dt + \tau_D \frac{d\epsilon(t)}{dt}\right\}$$

Taking laplace

$$G_c(s) = \frac{C(s)}{\epsilon(s)} = k_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

$\epsilon = y_s - y$

Drawback of PID

NOTES:

(i) If $\epsilon \to$ constant, then D-action will be zero.

(ii) For a noisy response (noisy y) with almost zero error D-term leads to large control action. (unnecessary)

$\tau_D \to$ Derivative time constant

W.C.R.

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#. Dynamic Behaviour of Closed Loop Process:

Block Diag. of closed loop system

Process: $y(s) = G_p(s)\, m(s) + G_d(s)\, d(s)$.

Meas. Device: $y_m(s) = G_m(s)\, y(s)$.

Controller: $E(s) = y_{sp}(s) - y_m(s)$ ---- Comparator

$C(s) = G_c(s)\, E(s)$ ---- Controller

F.C.E: $m(s) = G_f(s)\, C(s)$.

$$m(s) = G_f(s)\, G_c(s)\, E(s)$$

$$= G_f(s)\, G_c(s) \left[y_{sp}(s) - y_m(s) \right]$$

$$m(s) = G_f(s)\, G_c(s) \left[y_{sp}(s) - G_m(s)\, y(s) \right]$$

Now,

$$y(s) = G_p(s)\, m(s) + G_d(s)\, d(s)$$

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$$y = G_p G_f G_c [y_{sp} - G_m y] + G_d d$$

$$y = G_p G_c G_f y_{sp} - G_p G_c G_f G_m y + G_d d$$

$$\boxed{y = \frac{G_p G_c G_f}{(1 + G_p G_c G_f G_m)} y_{sp} + \frac{G_d}{(1 + G_p G_c G_f G_m)} d}$$

It ~~represent~~ provide effect of y_{sp} on y.

It provide effect of d on y.

..... closed loop T.F. (CLTF).

If $G_p G_c G_f = G$

then

$$y = \frac{G}{1 + G G_m} y_{sp} + \frac{G_d}{1 + G G_m} d \quad \ldots\ldots \checkmark$$

$$y = G_{sp} y_{sp} + G_{load} d$$

Where

$$G_{sp} = \frac{G}{1 + G G_m}$$

$$G_{load} = \frac{G_d}{1 + G G_m}$$

$$y = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} y_{sp} + \frac{G_d}{1 + G_p G_c G_f G_m} d$$

- Use this T.F. usually in the analysis of diff. features.

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- Control performance is usually investigated by performing 2 test, one is servo test & Regulatory test.

<u>Servo Test:</u> $d = 0$

$$y = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} \cdot y_{sp}$$

servo problem / set point tracking.

<u>Regulatory Test:</u> $y_{sp} = 0$

$$y = \frac{G_d}{1 + G_p G_c G_f G_m} \cdot d$$

Disturbance Rejection performance.

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Example → Formulation of closed Loop Diag. for a liquid level System:

Obj: h h_{ss}

Controlled Pair:

CV	MV	LV
h	f_o	f_i

DPC → Differential Press. Cell

Process:

$$A\frac{dh}{dt} = f_i - f_o$$

$$A s\, h(s) = f_i(s) - f_o(s)$$ ..... Laplace transform

$$h(s) = \frac{1}{As} f_i(s) - \frac{1}{As} f_o(s)$$ ..... liquid tank

~~General form~~

$$y(s) = G_p(s)\, m(s) + G_d(s)\, d(s)$$ ..... General form

$m \to f_o$ & $d \to f_i$

So. $G_p(s) = -\frac{1}{As}$, $G_d(s) = \frac{1}{As}$

Measuring Device:

$\Delta P \propto h$ $\qquad$ $\Delta P = \beta h$

Considering 2nd ord. dynamic of DPC.

$$\tau^2 \frac{d^2 h_m}{dt^2} + 2\tau\zeta \frac{dh_m}{dt} + h_m = k_p \Delta P = k_p \beta h$$

$$G_m(s) = \frac{h_m(s)}{h(s)} = \frac{k_p \beta}{(\tau^2 s^2 + 2\tau\zeta s + 1)}$$ ..... Laplace Transform

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Controller :

PI Controller ..... $G_c(s) = k_c\left(1 + \frac{1}{\tau_I s}\right)$

F.C.E :

Assuming 1st ord. dynamics of control valve.

$$G_f = \frac{k_v}{(\tau_v s + 1)}$$

$k_v \rightarrow$ Gain of control valve

$\tau_v \rightarrow$ Time constant of control valve.

Fig : Closed loop block diag. of liq. level system

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OFFSET

Offset is defined as the diff. b/w ultimate value of set point and ultimate value of respons.

$$= R(\infty) - C(\infty)$$
$$= Y_{sp}(\infty) - Y(\infty)$$

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offset is defined as steady state error exhibited by system due to which output is not able to match input.

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Second Order Sys. Under P-Only Controller:

Here we only consider servo problem.

Process: $G_p(s) = \dfrac{k_p}{\tau_p^2 s^2 + 2\tau_p \zeta s + 1} = \dfrac{Y(s)}{M(s)}$

Controller: $G_c(s) = k_c = \dfrac{C(s)}{E(s)}$ ..... P-only cont

F.C.E & M.D.: $G_f(s) = G_m(s) = 1$.

CLTF

$$Y(s) = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} Y_{sp}(s) + \frac{G_d}{1 + G_p G_c G_f G_m} D(s)$$

$$Y = \frac{G_p k_c}{1 + G_p k_c} Y_{sp}(s) + \frac{G_d}{1 + G_p k_c} D(s)$$

Servo Mode

$\cancel{Y_{sp}(s) = \frac{1}{s}}$ & $D(s) = 0$

$$Y(s) = \frac{\dfrac{k_p k_c}{\cancel{(\tau_p^2 s^2 + 2\tau_p \zeta s + 1)}}}{\dfrac{\tau_p^2 s^2 + 2\tau_p \zeta s + 1 + k_p k_c}{\cancel{(\tau_p^2 s^2 + 2\tau_p \zeta s + 1)}}} Y_{sp}(s)$$

$$Y(s) = \frac{k_p'}{\{(\tau_p')^2 s^2 + 2\tau_p' \zeta' s + 1\}} Y_{sp}(s)$$

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$$k_p = \frac{k_p k_c}{1+k_p k_c}, \quad \tau_p = \frac{\tau_p}{\sqrt{(1+k_p k_c)}}, \quad \zeta = \frac{\zeta}{\sqrt{1+k_p k_c}}$$

$$\zeta_p \zeta = \frac{\tau_p \zeta}{1+k_p k_c} \Rightarrow \frac{\tau_p \zeta}{\sqrt{1+k_p k_c}} \frac{\tau_p \zeta}{\sqrt{1+k_p k_c}} \Rightarrow \zeta' = \frac{\zeta}{\sqrt{1+k_p k_c}}$$

NOTES:

(1) Due to inclusion of P-only controller with 2nd order system, there is no change in the overall order of system.

(2) $\tau_p' < \tau_p \rightarrow$ Response become faster.

(3) $\zeta' < \zeta \rightarrow$ Under P-only controller we may obtained under-damped Response. (that means if we get ~~Overdamped~~ Critically damped response, then after introducing P-only controller we'll get underdamped response.)

Hence, response become more oscillatory.

Offset:

$$Y(s) = \frac{k_p'}{(\tau_p')^2 s^2 + 2\tau_p' \zeta' s + 1} Y_{sp}(s)$$

$Y_{sp}(t) = A$

$Y_{sp}(s) = A/s$

$$Y(s) = \frac{k_p'}{s\{(\tau_p')^2 s^2 + 2\tau_p' \zeta' s + 1\}}$$

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From F.V.T

$$Y(\infty) = A k_p'$$

$$\therefore \text{offset} = A Y_{sp}(\infty) - Y(\infty)$$

$$= A - A k_p'$$

$$\boxed{\text{offset} = A(1 - k_p')}$$

$$= A\left(1 - \frac{k_p k_c}{1 + k_p k_c}\right)$$

$$\boxed{\text{offset} = \frac{A}{1 + k_p k_c}}$$

as $k_c \to \infty \Rightarrow$ offset $\to 0$

- $\zeta' = \dfrac{\zeta}{\sqrt{1 + k_p k_c}} \Rightarrow$ As $k_c \uparrow \to \zeta' \downarrow$

  $k_c \downarrow \to \zeta' \uparrow$

(i) If we increase k_c, overall response become more oscillatory.

(ii) Larger the k_c value, smaller the offset.

Hence from dynamic behaviour, we observe that at higher k_c i.e. at k_{c_1} there is more oscillation & less offset & at lower k_c i.e. at k_{c_2} there is less oscillation & large offset.

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Effect of Integral Action :

Process : $G_p(s) = \dfrac{Y(s)}{M(s)} = \dfrac{k_p}{(\tau_p s + 1)}$ ---- 1st order process

Controller : $G_c(s) = \dfrac{C(s)}{E(s)} = \dfrac{k_c}{\tau_I s}$ ---- Integral Action

F.C.E & M.D : $G_f(s) = G_m(s) = 1$

C.L.T.F. :

$$Y(s) = \frac{G_c G_p G_f}{1 + G_p G_c G_f G_m} Y_{sp}(s) + \frac{G_d}{1 + G_p G_c G_f G_m} D(s)$$

Servo Mode $D(s) = 0$

$$Y(s) = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} Y_{sp}(s)$$

$$\cancel{Y(s) = \frac{k_p k_c}{\tau_p s + 1 + k_p k_c} Y_{sp}(s)}$$

$$Y(s) = \frac{\dfrac{k_p}{(\tau_p s + 1)} \dfrac{k_c}{\tau_I s}}{1 + \dfrac{k_p k_c}{\tau_I s (\tau_p s + 1)}} Y_{sp}(s)$$

$$Y(s) = \frac{k_p k_c}{\tau_I s (\tau_p s + 1) + k_p k_c} Y_{sp}(s)$$

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$$Y(s) = \frac{1}{(\tau_p')^2 s^2 + 2\tau_p' \zeta' s + 1}$$

Where

$$\tau_p' = \sqrt{\frac{\tau_I \tau_p}{k_p k_c}}, \quad \cancel{\tau_p' \zeta' = (\tau_p + \tau_I)\zeta / k_p k_c}$$

$$\cancel{\zeta = \tau_p + \tau_I}$$

$$2\tau_p' \zeta' = \frac{\tau_I}{k_p k_c}$$

$$\zeta' = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_p k_p k_c}}$$

↳ Value of τ_p' & ζ' depend on controller tuning Param. k_c & τ_I.

Offset:

~~Let Y_{sp} (~~ servo Mode →

$$Y_{sp}(t) = A \qquad D(s) = 0$$

$$Y_{sp}(s) = A/s$$

$$Y(s) = \frac{A}{s\{\tau_p'^2 s^2 + 2\tau_p' \zeta' s + 1\}}$$

From F.V.T

$$Y(\infty) = A$$

$$\therefore \quad \text{offset} = Y_{sp}(\infty) - Y(\infty)$$

$$= A - A$$

$$\boxed{\text{offset} = 0}$$

- Integral control action eliminate the offset.

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NOTES :

(1) Due to inclusion of I action, overall order of syst. changes (increases).

(2) $\zeta' \propto \sqrt{\frac{\tau_I}{\tau_p k_p k_c}}$ → So closed loop sys. provide any of the response depending on value of "k_c" & "τ_I".

(3) With ↑ of k_c → ζ' ↓

So the overdamped response move to under-damped response with large overshoot & Decay ratio.

(4) With ↓ in τ_I → ζ' ↓.

Closed Loop Behaviour

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#. EFFECT OF DERIVATIVE CONTROL ACTION :

Process: $G_p(s) = \dfrac{Y(s)}{M(s)} = \dfrac{k_p}{\tau_p s + 1}$ - - - - 1st Order Sys.

Controller: $G_c(s) = \dfrac{C(s)}{E(s)} = k_c \tau_D s$ - - - - - D-Action.

F.C.E. & M.D: $G_f(s) = G_m(s) = 1$

CLTF:

$$Y(s) = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} Y_{sp}(s) + \frac{G_d}{1 + G_p G_c G_f G_m} D(s)$$

Servo Mode: $D(s) = 0$

$$Y(s) = \frac{G_p G_c G_f}{1 + G_p G_c G_f G_m} Y_{sp}(s)$$

$$Y(s) = \frac{\dfrac{k_p}{(\tau_p s + 1)} k_c \tau_D s}{1 + \dfrac{k_p k_c \tau_D s}{(\tau_p s + 1)}} Y_{sp}(s)$$

$$Y(s) = \frac{k_p k_c \tau_D s}{\tau_p s + 1 + k_p k_c \tau_D s} Y_{sp}(s)$$

$$Y(s) = \frac{k_p'}{\tau_p' s + 1} Y_{sp}(s)$$

~~Where $k_p' = \dfrac{k_p k_c \tau_D s}{1 + k_p k_c \tau_D s}$ & $\tau_p' = \tau_p$~~

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$\tau_p' = (\tau_p + k_p k_c \tau_D)$ $\qquad k_p' = k_p k_c \tau_D s$

effective time constant.

NOTES:

(1) Due to inclusion of D-action, there is no change in overall order.

(2) $\tau_p' > \tau_p \rightarrow$ Response time increases i.e. Overall response become sluggish.

(3) With $\uparrow$ of $k_c \longrightarrow \tau_p' \uparrow$.

(4) With $\uparrow$ of $\tau_D \longrightarrow \tau_p' \uparrow$.

} i.e. Overall response become slower and $\uparrow$ in τ_D & k_c (Sluggishness increases)

(5) D-action does not eliminate the offset.

OFFSET: $\quad Y_{sp}(\infty) = Y(\infty)$

$$Y(s) = \frac{k_p k_c \tau_D s}{(\tau_p' s + 1)} Y_{sp}(s)$$

for $Y_{sp}(t) = A \longrightarrow Y_{sp}(s) = A/s$

$$Y(s) = \frac{A}{\cancel{s}} \frac{k_p k_c \tau_D \cancel{s}}{(\tau_p' s + 1)} \qquad Y(\infty) = \lim_{s \to 0} s\,Y(s)$$

$$Y(\infty) = \cancel{A k_p k_c \tau_D}\ 0$$

$$\therefore \text{offset} = A - \cancel{A k_p k_c \tau_D}\ 0$$

$$\boxed{\text{offset} = A\,(\cancel{1 - k_p k_c \tau_D})}$$

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. Determine the transfer function $H(s)/Q(s)$ for the liquid-level system shown in Fig. P6–1. Resistances R_1 and R_2 are linear. The flow rate from tank 3 is maintained constant at b by means of a pump; i.e., the flow rate from tank 3 is independent of head h. The tanks are noninteracting.

FIGURE P6–1

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Find the resulting concentration in each tank as a function of time.

4. (*a*) Find the transfer functions H_2/Q and H_3/Q for the three-tank system shown in Fig. P6-4 where H_2, H_3, and Q are deviation variables. Tank 1 and tank 2 are interacting.

(*b*) For a unit-step change in q (that is, $Q = 1/s$), determine $H_3(0)$ and $H_3(\infty)$, and sketch $H_3(t)$ versus t.

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6.6. In the two-tank mixing process shown in Fig. P6–6, x varies from 0 lb salt/ft^3 to 1 lb salt/ft^3 according to a step function. At what time does the salt concentration in tank 2 reach 0.6 lb salt/ft^3? The holdup volume of each tank is 6 ft^3.

FIGURE P6–6

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7.1. A step change of magnitude 4 is introduced into a system having the transfer function

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

Determine

(*a*) Percent overshoot

(*b*) Rise time

(*c*) Maximum value of $Y(t)$

(*d*) Ultimate value of $Y(t)$

(*e*) Period of oscillation

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7.15. A step change of magnitude 3 is introduced into the transfer function

$$\frac{Y(s)}{X(s)} = \frac{10}{2s^2 + 0.3s + 0.5}$$

Determine the overshoot and the frequency of oscillation.

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7.11. In the liquid-level system shown in Fig. P7–11, the deviation in flow rate to the first tank is an impulse function of magnitude 5. The following data apply: $A_1 = 1\ \text{ft}^2$, $A_2 = A_3 = 2\ \text{ft}^2$, $R_1 = 1$ ft/cfm, and $R_2 = 1.5$ ft/cfm.

FIGURE P7–11

(*a*) Determine expressions for $H_1(s)$, $H_2(s)$, and $H_3(s)$ where H_1, H_2, and H_3 are deviations in tank level for tanks 1, 2, and 3.

(*b*) Sketch the responses of H_1 (t), H_2 (t), and H_3 (t). (You need show only the shape of the responses; do not plot.)

(*c*) Determine H_1 (3.46), H_2 (3.46), and H_3 (3.46). For H_2 and H_3, use graphs in Chap. 7 of this text after first finding values of τ and ζ for an equivalent second-order system.

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PAGE

'998 (2-23) In a closed loop system, process to be controlled is an integrating process with T.F. $1/2s$. The controller proposed to be used is an integral controller with T.F. $1/T_I s$, when a step change in set point is applied to such a closed loop system, the controlled variable will exhibit

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2015 (61) The block diag. for a process with feedback control for output deviation variable 'h' is shown in fig. below. All T.F. are given with power factor of 'S' in minutes. A unit step change is made in the set point at $t=0$. The time req. for h to reach 50% of its ultimate value in min.

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